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AN ANALYTIC MODEL OF THE HEAT-INDUCED TROPICAL CIRCULATION IN THE PRESENCE OF A MEAN WIND.

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A linear analytic model of the response of the tropical troposphere - in the presence of a uniform zonal mean wind-to an isolated region of deep heating is presented. The O(1) components of the response are the long Kelvin and planetary waves, as found in earlier work by Matsuno, Gill, and others. The behaviour of these modes is a function of a non-dimensional parameter which combines the damping time, the wave speed, the mean wind speed, and the length of the heating region. It is interpreted as the ratio of the heating region length to the wave decay distance. When this ratio is small the response consists of long waves propagating away from the heating region. But when it is large, disturbances are locally determined at each longitude, and the dynamics are those of two-dimensional Hadley circulations.

In a westerly mean wind one also finds short stationary mixed and planetary waves downstream of the heating region. Their amplitude is small compared to the long waves, and proportional to U/L_H^2 for small U, where U is the mean wind speed wind and L_H the length of the heating region. In easterly mean winds the short stationary waves are replaced by inertial boundary layers at the edges of the heating region.

1. Introduction

Over the past twenty years much progress has been made with quantitative models to help understand the flow associated with deep heating of the tropical atmosphere. In one approach one stresses the fact that the heating is distributed in a band of latitudes that circles the earth. As a convenient approximation one then uses a zonally symmetric model, as in Dickinson (1971 a,b), for example. These may be thought of as quantitative models of ideas of the tropical general circulation going back to Halley and Hadley.

Another approach has stressed that the winds in the tropics are by no means zonally symmetric, and that the heating is largely concentrated in a few regions such as Brazil, or the maritime continent of South East Asia. In this approach the natural starting point for simple analytic models is to consider an isolated heating region in an atmosphere otherwise at rest. The models of Matsuno (1966), Webster (1972), Gill (1980), Heckley and Gill (1984) have shown how the flow can be described in terms of thermally forced, long, and equatorially trapped waves. Usually these studies have used a heating region of roughly circular shape and a diameter of around thirty degrees, which roughly corresponds to the shape of the first wave mode. This approach has recently been generalised by models which allow for a uniform zonal wind. Thus Lau and Lim (1982) have studied the effects of the mean wind on the long waves, whilst Lim and Chang (1983) and Lau and Lim (1984) also allow for stationary short planetary waves, but unfortunately without simple analytical results for the amplitudes, decay rates, etc. While in practice the zonal mean winds may be difficult

to evaluate, as the heat-induced flows cover a good fraction of the earth, they will undoubtedly have some shear in the horizontal and in the vertical. Nevertheless, the effect of the barotropic part of the mean flow will be important, and as moreover this is the easiest case to tackle analytically, it is a useful way of extending our understanding of heat-induced flows.

Our aim, then, is to use as simple a model as possible to study the response of the tropical atmosphere to heating distributions of various shapes in the presence of a mean wind. Moreover, as the role and modelling of the damping processes is poorly understood, we shall model them simply, but examine a range of damping time constants. These three parameters, the shape of the heating, the mean wind, and the damping time, which may seem unrelated, will be found to combine in a single non-dimensional parameter governing the response for each mode. When this parameter, which may be interpreted as the ratio of the length of the heating region to the wave decay distance, is small the response is wavelike, as in Gill (1980). On the other hand, when it is large the response is dynamically similar to the zonally symmetric model of Dickinson (1971a). The zonally symmetric model and the wave model appear to be two extremes, or limiting cases, with the response to any given heating distribution falling somewhere in between, depending on the different parameters.

2. The Model

The reason for the simplicity of the model is that the problem is made separable in x, y and z. This is made possible by the relatively simple vertical structure of of the heating associated with deep convection. Whilst the convection process is a complex subject itself (Houze and Betts 1981), the vertical structure of the resulting heating is fairly well established: it is strongest in the middle troposphere and becomes weak as one approaches the surface or the tropopause (Houze, 1982; Hantel and Baader, 1978). It thus resembles a first baroclinic mode, and the assumption made here is that the deviation of the heating function from a first baroclinic mode is small enough for this to be of secondary importance as far as the response is concerned.

As we are interested in the tropospheric response to a specified heating function we put a rigid lid at the tropopause, and we assume that the heating and the response have a first baroclinic mode structure. Hartmann, Hendon and Houze (1984) have used different vertical heating distributions, with significant changes in the vertical structure of the response, but only small changes in its horizontal structure. Silva Dias and Bonatti (1985) have studied the vertical mode structure of the summer circulation observed over tropical South America. They find that "as the equator is approached, an internal mode with equivalent depth of the order of 250m becomes dominant". This is the mode we are considering.

The equations are linearized about a mean zonal wind U. The fluid is taken to be in hydrostatic balance, incompressible and uniformly stratified. The equations are non-dimensionalised as in Gill (1980), giving

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) u - \frac{1}{2} y v = -\frac{\partial \phi}{\partial x} \\
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) v + \frac{1}{2} y u = -\frac{\partial \phi}{\partial y} \\
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) \Theta + w = Q \\
&\partial\phi/\partial z = \Theta \\
&\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0
\end{aligned} \tag{1}$$

where now the surface is at z = 0 and the tropopause is at $z = \pi$. The parameter Δ is the non-dimensional mean wind, $\Delta = U/C$, $C = NH$, where C is the speed of long internal gravity waves ($C \sim 50$m/s), N is the buoyancy frequency ($N \sim 10^{-2}$ s^{-1}), and H the vertical scale ($H \sim 4$km). The horizontal scale is the equatorial Rossby radius $a = (C/2\beta)^{1/2}$ ($a \sim 1000$km ~ 10deg).

We now introduce the vertical mode structure and eliminate w and Θ in (1). With

$$\begin{aligned}
(u, v, \phi) &= (u_*, v_*, \phi_*) \cos z \\
(w, \Theta, Q) &= (w_*, \Theta_*, Q_*) \sin z
\end{aligned} \tag{2}$$

equations (1) reduce to

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) u_* + \frac{\partial \phi_*}{\partial x} - \frac{1}{2} y v_* = 0 \\
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) \phi_* + \frac{\partial u_*}{\partial x} + \frac{\partial v_*}{\partial y} = -Q_* \\
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) v_* + \frac{1}{2} y u_* = -\frac{\partial \phi_*}{\partial y}
\end{aligned} \tag{3}$$

where all the dependent variables are functions of x, y, and t only. To make the problem separable in y we work with new dependent variables q and

$$q_* = \phi_* + u_* \quad , \quad r_* = \phi_* - u_* \tag{4}$$

In terms of q_*, r_*, and v_*, and after dropping the asterisks, equations (3) become

$$\begin{aligned}
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) q + \frac{\partial q}{\partial x} + \left(\frac{\partial v}{\partial y} - \frac{1}{2} y v\right) = -Q \\
&\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) r - \frac{\partial r}{\partial x} + \left(\frac{\partial v}{\partial y} + \frac{1}{2} y v\right) = -Q \\
&2\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) v + \left(\frac{\partial q}{\partial y} + \frac{1}{2} y q\right) + \left(\frac{\partial r}{\partial y} - \frac{1}{2} y r\right) = 0
\end{aligned} \tag{5}$$

Note that in the y momentum equation, the last of (5), we keep the v term. By not making the long wave approximation we allow the possibility that in a westerly mean wind short stationary planetary waves will be generated.

Equations (5) have solutions in terms of meridional normal modes with the structure

$$q = q_{n+1}(x,t)\, D_{n+1}(y), \quad v = v_n(x,t)\, D_n(y), \quad r = r_{n-1}(x,t)\, D_{n-1}(y) \tag{6}$$

where the $D_n(y)$ are parabolic cylinder functions. These functions satisfy the relationships

$$\frac{dD_n}{dy} + \frac{1}{2} y D_n = n D_{n-1} \quad , \quad \frac{dD_n}{dy} - \frac{1}{2} y D_n = -D_{n+1} \tag{7}$$

and the first few functions are

$$D_0, D_1, D_2, D_3 = \left(1, y, y^2 - 1, y^3 - 3y\right) \exp\left(-\frac{1}{4} y^2\right) \tag{8}$$

The horizontal distribution of the heating is taken to be separable and of

the form

$$Q = F(x) \sum_{n=0}^{\infty} Q_n D_n(y) \tag{9}$$

We can now use (6) - (9) in (5) to obtain the equations for all the modes.

(i) The Kelvin mode (n = -1)

In (6) the lowest value for n is n = -1, in which case v = r = 0. this mode, conventionally called the Kelvin mode, is governed by the single equation

$$\left(\frac{\partial}{\partial t} + \Delta \frac{\partial}{\partial x} + \epsilon\right) q_0 + \frac{\partial q_0}{\partial x} = -Q_0 F(x) \tag{10}$$

from the first of (5).

(ii) The Mixed mode (n = 0)

With n = 0 in (6) we have q, v $\neq$ 0, r = o, and the equations are

$$\begin{aligned} &\left(\frac{\partial}{\partial t} + \Delta \frac{\partial}{\partial x} + \epsilon\right) q_1 + \frac{\partial q_1}{\partial x} - v_0 = -Q_1 F(x) \\ &2\left(\frac{\partial}{\partial t} + \Delta \frac{\partial}{\partial x} + \epsilon\right) v_0 + q_1 = 0 \end{aligned} \tag{11}$$

In the absence of a mean wind one can make the long wave approximation (Gill, 1980), in which case the second of (11) reduces to $q_1 = 0$, and therefore $v_0 = Q_1$.

(iii) The Gravity and Planetary modes (n $\geq$ 1)

Now q, v, and r are all non-zero, and the equations are

$$\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) q_{n+1} + \frac{\partial q_{n+1}}{\partial x} - v_n = -Q_{n+1} F(x)$$
$$\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) r_{n-1} - \frac{\partial r_{n-1}}{\partial x} + n v_n = -Q_{n-1} F(x) \qquad (12)$$
$$2\left(\frac{\partial}{\partial t} + \Delta\frac{\partial}{\partial x} + \epsilon\right) v_n + (n+1) q_{n+1} - r_{n-1} = 0$$

We note that as we go from the Kelvin mode to the mixed mode, and on to the planetary modes we go from a first order system, to a second order system, and on to a third order system of equations. As we shall see, they have one, two and three types of solutions respectively. For simplicity we now solve them in this same order.

3. The Kelvin Mode

To solve (10) for the Kelvin mode we need to specify the horizontal distribution of the heating. We shall take

$$Q = I\alpha K (1-\cos kx)\, D_0[\alpha(y-y_0)] \quad , \quad 0 < x < 2\pi/K \tag{13}$$

with as parameters I, the total amount of heat, α, the width of the heating region, $2\pi/K$, the length of the heating region, and y_0, the latitude of maximum heating. We show in the appendix how $D_0[\alpha(y-y_0)]$ can be expressed as a series in $D_n(y)$,

$$D_0[\alpha(y-y_0)] = \sum_{n=0}^{\infty} c_n D_n(y)$$

$$c_{n+1} = \frac{1}{n+1}\left[\left(\frac{1-\alpha^2}{1+\alpha^2}\right)c_{n-1} + \left(\frac{\alpha^2}{1+\alpha^2}\right)y_0 c_n\right], \quad c_0 = \left(\frac{2}{1+\alpha^2}\right)^{1/2} \exp\left(\frac{-1}{4}\frac{\alpha^2}{1+\alpha^2}y_0^2\right) \tag{14}$$

The heating function can thus be written in the separable form (9) where we take

$$F(x) = K(1-\cos kx) \quad , \quad Q_n = I\alpha c_n \quad \text{for } 0 < x < 2\pi/K \tag{15}$$

Taking F(x) in this form ensures that the coefficients Q_n do not change as a given amount of heating is spread over different lengths.

A particular solution of (10) for the heating region is then

$$q_0 = \left(\frac{-1}{\gamma_{-1}} + \frac{\gamma_{-1}}{1+\gamma_{-1}^2}\cos kx + \frac{1}{1+\gamma_{-1}^2}\sin kx\right)\frac{Q_0}{1+\Delta} \quad , \quad \gamma_{-1} = \epsilon/(1+\Delta)K \tag{16}$$

in the steady state. The subscript -1 is used to denote the Kelvin mode.

We note that $(1+\Delta)$ is the nondimensional form of (C+U), where C is the Kelvin wave speed and U the zonal mean wind speed. Since C $\sim$ 50m/s and U$\sim$10m/s or less, we can make the assumption $\Delta \ll 1$. The particular solution (16) is then mainly a function of the non-dimensional number γ which combines a number of the physical parameters in the model: the wave speed, the mean wind, the length of the heating region, and the damping time constant. The interpretation of this number will become clear after we complete the solution for the Kelvin mode.

The free solution of (10) is simply

$$q_0 = A_{-1} \exp\left[-\gamma_{-1} K (x+\xi)\right] \tag{17}$$

which is an eastward decaying long wave. The boundary conditions are that q_0 must be continuous at x=0 and at $x=2\pi/K$. Since $q_0 = 0$ for $x<0$ we add a free solution for $x>0$ with

$$A_{-1} = \frac{1}{\gamma_{-1}(1+\gamma_{-1}^2)} \frac{Q_0}{1+\Delta} \quad , \quad \xi = 0 \tag{18}$$

To satisfy the boundary condition at $x=2\pi/K$ we must take another free solution (17) for $x>2\pi/K$, but with amplitude $-A_{-1}$ and $\xi = -2\pi/K$.

We can now see the meaning of γ. If we denote by Lw the distance over which the Kelvin wave decays then from (17) $\gamma K L_w \sim \pi$. But π/K is a measure of the length of the heating region, L_H, so that

$$\gamma \sim L_H / L_w \tag{19}$$

Thus γ is the ratio of the length of the heating region to the distance over which waves decay. While there is some debate as to the most suitable value of ϵ, typically $\epsilon \ll 1$, $K \sim O(1)$, so that for the Kelvin mode $\gamma \ll 1$.

For small γ, q in the heating region is approximately

$$q_o \simeq (-kx + \sin kx)\frac{Q_o}{1+\Delta} = \frac{Q_o}{1+\Delta}\int_0^x F(x')\,dx' \qquad (20)$$

Thus in the frictionless limit the Kelvin mode flow in the heating region is a function of Kx only, and is proportional to the integral along the propagation path of the heating function F(x). This is a general result for the frictionless limit in the heating region (c.f. Gill and Phlips 1986). The strength of the Kelvin mode flow at the eastern end of the heating region is then independent of the length of the heating region, and inversely proportional to $1+\Delta$. But since typically Δ is small, the Kelvin mode is only weakly affected by the mean wind.

4. The Mixed Mode

The mixed mode is excited by the lowest antisymmetric mode heating. Unlike the Kelvin and planetary modes, the mixed mode has no solutions with zero frequency but finite phase and group velocities, so that the response in still air is just local forced response. But in the presence of a mean wind a wider variety of solutions is possible.

To solve for the forced response in the heating region we reduce (11) to a single equation for v_0 and neglect terms $O(\Delta\partial/\partial x + \epsilon)^2$ .In the steady state

$$2\frac{\partial}{\partial x}\left(\Delta\frac{\partial}{\partial x}+\epsilon\right)v_0 + v_0 = k(1-\cos kx)\,Q_1 \tag{21}$$

a particular solution of which is

$$v_0 = \left(1-2k^2\Delta - \frac{1}{1+\gamma_0^2}\cos kx + \frac{\gamma_0}{1+\gamma_0^2}\sin kx\right)\frac{kQ_1}{(1-2k^2\Delta)} \tag{22}$$

where $\gamma_0 = \dfrac{-2\epsilon k}{1-2k^2\Delta} = \dfrac{\epsilon}{\left(-1/2k^2+\Delta\right)k}$

and $-1/2k^2$ is the phase speed of mixed mode waves in the low frequency approximation. But as $\epsilon, \Delta \ll 1$ and $K \sim O(1)$, then $\gamma_0 \ll 1$ so that as a first approximation

$$v_0 \simeq k(1-\cos kx)\,Q_1 = F(x)\,Q_1 \quad , \quad q_1 \simeq 0 \tag{23}$$

which is the long wave solution. To the next level of approximation the

particular solution is

$$v_0 \simeq \left[(1-\cos kx) - 2k^2\Delta + \gamma_0 \sin kx \right] \frac{kQ_1}{(1-2k^2\Delta)}$$

$$q_1 \simeq -2\left[\epsilon(1-\cos kx) + k\Delta \sin kx \right] \frac{kQ_1}{(1-2k^2\Delta)} \qquad (24)$$

where q_1 has been obtained by substituting for v_0 in the second of (11). At this order v_0 is no longer zero at the edges of the heating region, so free solutions of (21) will be needed to make the flow continuous.

If we look for free solutions of (21) of the form

$$v_0 = e^{\alpha x} \qquad (25)$$

we obtain the quadratic equation

$$2\Delta\alpha^2 + 2\epsilon\alpha + 1 = 0 \qquad (26)$$

To keep the algebra to a minimum we shall now assume that the mean wind is not too weak, formally $|\Delta|^{1/2} >> \epsilon$. When Δ becomes very small the structure of the free solutions changes rapidly, but this is unimportant as in that limit their amplitude becomes very small too. With this assumption the roots of (26) become

$$\text{for } \Delta > 0 \qquad \alpha = \pm i(2\Delta)^{-1/2} - \epsilon/2\Delta = \pm i k_0 - \epsilon/2\Delta \qquad (27)$$

$$\text{for } \Delta < 0 \qquad \alpha = \pm(-2\Delta)^{-1/2} = \pm k_0' \qquad (28)$$

Thus a positive (westerly) mean wind gives rise to eastward decaying

waves, while a negative (easterly) mean wind gives rise to strongly damped exponential solutions. This is the equatorial equivalent of the problem studied by Moore (1963) for mid-latitudes. Moore finds stationary planetary waves in westerly mean flows, and inertial boundary layers in easterly mean flows.

The stationary damped wave (27) may readily be understood in terms of simple waves propagating in still air. From (11) the equation for low frequency mixed mode waves is

$$2\left(\frac{\partial}{\partial t} + \epsilon\right)\frac{\partial v_0}{\partial x} + v_0 = 0 \qquad (29)$$

If we look for free solutions with spatial decay of the form

$$v_0 = \exp i(kx - \omega t) \qquad (30)$$

where ω is real and $k = k_r + ik_i$, $k_i/k_r = O(\epsilon)$, then from the O(1) equation

$$\omega = -1/2\, k_r \quad \Rightarrow \quad c = -1/2\, k_r^2 \,, \quad c_g = 1/2\, k_r^2 \qquad (31)$$

and from the $O(\epsilon)$ equation

$$k_i = 2k_r^2\epsilon = \epsilon/c_g \qquad (32)$$

In the presence of a mean wind Δ the waves will be stationary when the phase speed is zero,

$$\Delta = -c = 1/2k^2 \quad \Rightarrow \quad k = (2\Delta)^{-1/2} \tag{33}$$

as in (27). The group velocity relative to the ground is then

$$c_g = 1/2k^2 + \Delta = 2\Delta \tag{34}$$

or just twice the mean wind speed. The spatial damping rate is them $-\epsilon/2\Delta$, as found in (27).

We note that when $\Delta = 0$ there is another well-known solution to (26). This is $\alpha = -1/2\epsilon$, which corresponds to a Stommel boundary layer. This solution will not be needed here, since when $\Delta = 0$ the particular solution satisfies $v = q = 0$ at the edges of the heating region.

To summarize, for $\Delta > 0$ the stationary mixed waves take the form

$$v_0 = L \cos k_0(x+\xi) \exp\left[-\epsilon(x+\xi)/2\Delta\right], \quad q_1 = -2\Delta\, \partial v_0/\partial x \tag{35}$$

and since $\Delta \ll 1$, q_1 will be much smaller than v_0. Since this wave propagates eastward we have $v_0 = q_1 = 0$ for $x < 0$. The boundary conditions at $x = 0$ are then (24) + (35) = 0 at $x = 0$, and therefore

$$L = 2k^2\Delta \frac{kQ_1}{1-2k^2\Delta}, \quad \xi = 0 \tag{36}$$

The flow must also be continuous at $x = 2\pi/K$, so for $x > 2\pi/K$ we add another free solution (35) but with amplitude $-L$ and $\xi = -2\pi/K$. The amplitude of the stationary waves is $O(2K^2\Delta)$ compared to the flow in the heating region, and thus rapidly becomes unimportant if the heating region becomes

long. Moreover, the stationary wave pattern downstream of the heating region will not grow monotonically in amplitude as is increased. As the wavenumber k is also a function of Δ there will be periodic reinforcement and cancellation of the waves.

For easterly mean winds we have two inertial boundary layers,

$$v_0 = M \exp k_0'(x+\xi) \quad , \quad q_1 = v_0 / k_0' \tag{37}$$

$$v_0 = N \exp -k_0'(x+\xi) \quad , \quad q_1 = -v_0 / k_0' \tag{38}$$

To satisfy the boundary conditions at x=0 we use (37) for x<0 and (38) for x>0. Since q_1 must be continuous at x=0 and the forced part of q_1 is zero at x=0 we have M=-N. Then to make v_0 continuous at x=0 we use (24) and find

$$M = -N = -k^2\Delta \frac{kQ_1}{1-2k^2\Delta} \quad , \quad \xi = 0 \tag{39}$$

To make the flow continuous at $x=2\pi/K$ we use (37) - (38) again, but with $\xi = -2\pi/K$ and the signs of M and N changed.

5. The Planetary Modes

We can reduce equations (12) to two equations for q and Ω,

$$\mathcal{L}(q_{n+1}) = \left[1+\varepsilon\frac{\partial}{\partial x}\left(\Delta\frac{\partial}{\partial x}+\epsilon\right)\right]\left(Q_{n-1}+nQ_{n+1}\right)F(x) - \varepsilon\frac{\partial}{\partial x}\left(\Delta\frac{\partial}{\partial x}+\epsilon\right)\left[Q_{n-1}+(n+1)Q_{n+1}\right]F(x)$$

$$\mathcal{L}(\Omega_{n-1}) = (n+1)\left[1+\varepsilon\frac{\partial}{\partial x}\left(\Delta\frac{\partial}{\partial x}+\epsilon\right)\right]\left(Q_{n-1}+nQ_{n+1}\right)F(x) - \varepsilon n\frac{\partial}{\partial x}\left(\Delta\frac{\partial}{\partial x}+\epsilon\right)\left[Q_{n-1}+(n+1)Q_{n+1}\right]F(x) \quad (40)$$

$$\text{where } \mathcal{L}(q) = \left[1-\left(\varepsilon n+1-\varepsilon\frac{\partial^2}{\partial x^2}\right)\Delta\right]\frac{\partial q}{\partial x} - \left(\varepsilon n+1-\varepsilon\frac{\partial^2}{\partial x^2}\right)\epsilon q \quad (41)$$

in the steady state. The solution of these equations is simpler if we look at their properties first. We note that there is a good deal of symmetry between equations (40). If we make the long wave approximation in the third equation (12) then instead of (40) and (41) we obtain

$$\left[1-(\varepsilon n+1)\Delta\right]\frac{\partial q_{n+1}}{\partial x} - (\varepsilon n+1)\epsilon q_{n+1} = \left(Q_{n-1}+nQ_{n+1}\right)F(x) \quad (42)$$

$$\Omega_{n-1} = (n+1)q_{n+1} \quad (43)$$

Comparing (42) and (43) to (40) and (41) suggests that the first forcing term in each of the equations (40) is responsible for generating the long waves. This in turn suggests that it is the second forcing term in these equations which is forcing any short stationary waves or inertial boundary layers. These ideas may be put on a firm basis by looking at the properties of the free modes.

If we look for free solutions of (40) of the form

$$\mathcal{L}(q) = 0 \quad , \quad q = e^{\alpha x} \quad (44)$$

we obtain the cubic equation

$$2\Delta\alpha^3 + 2\epsilon\alpha^2 + \left[1-(2n+1)\Delta\right]\alpha - (2n+1)\epsilon = 0 \qquad (45)$$

From the results for the mixed mode and our knowledge of the dispersion relation for waves in still air we expect that one root will be small, and correspond to long planetary waves, and that the two others will be large and correspond to short stationary waves or inertial boundary layers. (There will be no stationary gravity waves as $\Delta << 1$). We therefore look for approximate solutions of (45) with $\alpha << 1$ and $\alpha >> 1$.

If we assume that $\alpha << 1$ then we find approximately

$$\alpha = \frac{(2n+1)\epsilon}{1-(2n+1)\Delta} = \frac{-\epsilon}{\frac{-1}{2n+1}+\Delta} = -\gamma_n^* k \qquad (46)$$

$$\text{and } v_n = \frac{2n+2}{1-(2n+1)\Delta}\epsilon q_{n+1} = -\left(\frac{2n+2}{2n+1}\right)\gamma_n^* k\, q_{n+1}, \quad \zeta_{n-1} = (n+1)\, q_{n+1}$$

where the superscript * indicates that the long wave approximation has been made. As expected we have a slow decay proportional to ϵ, and inversely proportional to the group velocity of long planetary waves. Note, however, that if n becomes large or $\Delta \dashrightarrow 1/(2n+1)$ then the approximation $\alpha << 1$ breaks down. But these are the conditions under which γ becomes large, suggesting that for large γ the wave approach is unsuitable because the energy cannot propagate. If $\Delta > 1/(2n+1)$ the waves decay eastward.

If we now assume that $\alpha >> 1$, and once again that $|\Delta|^{1/2} >> \epsilon$, so that we ignore the complications that arise when the mean wind is very weak, then we find approximately

$$\text{for } \Delta>0 \qquad \alpha = \pm i\left[\frac{1-(2n+1)\Delta}{2\Delta}\right]^{1/2} - \frac{\epsilon}{2\Delta} = \pm i\, k_n - \epsilon/2\Delta \tag{47}$$

$$\text{for } \Delta<0 \qquad \alpha = \pm\left[\frac{1-(2n+1)\Delta}{-2\Delta}\right]^{1/2} = \pm k'_n \tag{48}$$

as long as $(2n+1)\Delta<<1$. Thus, as expected, we find damped stationary waves in a westerly flow, and inertial boundary layers in an easterly flow. Note, however, the qualification $(2n+1)\Delta<<1$, which is equivalent to saying that γ may not be large.

For solutions with a short zonal scale, $\alpha>>\epsilon$, the expressions for v_n and r_{n-1} in terms of q_{n+1} are approximately

$$v_n = \partial q_{n+1}/\partial x \;, \quad r_{n-1} = n\, q_{n+1} \tag{49}$$

from (12) when $\Delta<<1$. Now π is approximately n times q, whereas for the long waves (46) it is (n+1) times q.

We can easily check that (47) has the properties of stationary planetary waves. Without going through the calculations (see Gill 1982), we shall have stationary waves when the phase speed is zero,

$$\Delta = -\frac{\omega}{k} = \frac{1}{2n+1+2k^2} \qquad \Rightarrow \quad k = \left[\frac{1-(2n+1)\Delta}{2\Delta}\right]^{1/2} \tag{50}$$

as found in (47). If we compute the group velocity we find that

$$c_g = 2\Delta\left[1-(2n+1)\Delta\right] \tag{51}$$

For small Δ the group velocity is twice the mean wind speed, but as Δ --->1/(2n+1) (γ becoming large) the group velocity goes to zero, i.e. disturbances do not propagate. Note that the linearised form of (51) is consistent with the decay rate in (47).

Returning now to the forced problem, we see by comparing the ratio of the forcing terms in the equations (40) with the ratio n/q for the long and short waves that the first forcing terms indeed correspond to the long wave component, and the second to short waves or inertial boundary layers. To solve for the long waves with a minimum of algebra we wish to neglect any terms which make only a small quantitative contribution to the solution. This is in fact what we do when we make the long wave approximation. We shall therefore use (42) and (43) for the long wave component. In the heating region we then find

$$q_{n+1} = \left(\frac{1}{\gamma_n^*} - \frac{\gamma_n^*}{1+\gamma_n^{*2}} \cos kx - \frac{1}{1+\gamma_n^{*2}} \sin kx \right) \frac{Q_{n-1} + n Q_{n+1}}{1 + \Delta / C_n^*} \tag{52}$$

$$v_n = \left[Q_{n-1} + (n+1) Q_{n+1} \right] F(x) + \frac{2(n+1)}{1+\Delta/C_n^*} \left[\epsilon q_{n+1} + \Delta \left(Q_{n-1} + n Q_{n+1} \right) F(x) \right] \tag{53}$$

where $C_n^* = -1/(2n+1)$

When n becomes large, the heating region becomes long, or $\Delta \dashrightarrow 1/(2n+1)$ there will be wave modes for which $\gamma_n^{*2} >> 1$ (the formal "large γ" criterion). Equation (52) may then be approximated by

$$q_{n+1} \simeq \frac{C_n^*}{\epsilon} \left(Q_{n-1} + n Q_{n+1} \right) F(x) \tag{54}$$

All variables are then proportional to the heating at the same longitude. The amplitudes are largest in the middle of the heating region and become very small at its edges. That the free wave solutions become invalid in this limit is thus not a problem: there is no need for them, the flow is simply proportional to the heating at the longitude in question.

When $\gamma \sim O(1)$ or less we complete the solution (52), (53) by matching at the edges of the heating region with long waves

$$q_{n+1} = A_n \exp\left[-\gamma_n^* K (x+\xi)\right] \tag{55}$$

and (46). Consider first the matching at x = 0. There are two cases to consider. If $\Delta < 1/(2n+1)$ the wave decays westward and A_n must be equal to the value of (52) at x = 0,

$$A_n = \frac{1}{\gamma_n^*(1+\gamma_n^{*2})} \frac{Q_{n-1} + n Q_{n+1}}{1 + \Delta/c_n^*} \quad , \quad \xi = 0 \tag{56}$$

If $\Delta > 1/(2n+1)$ the long waves decay eastward, so that q = 0 for x < 0 and takes minus the value in (56). To satisfy the boundary conditions at x = $2\pi/K$ we need another solution (55), but with $\xi = -2\pi/K$ and minus the amplitude at x = 0.

In the frictionless limit, when $\gamma_n^{*2} \ll 1$, q_{n+1} again takes a simple form in the heating region,

$$q_{n+1} \simeq \left[(Kx - 2\pi) - \sin Kx\right] \frac{Q_{n-1} + n Q_{n+1}}{1 + \Delta/c_n^*} = \frac{Q_{n-1} + n Q_{n+1}}{1 + \Delta/c_n^*} \int_{2\pi/K}^{x} F(x')\, dx' \tag{57}$$

Now q_{n+1} increases smoothly from 0 at $x = 2\pi/K$ to some value which is independent of K at the western edge of the heating region. Note the similarity with expression (20) for the Kelvin mode in the heating region: in both cases the amplitude is proportional to the integral of the heating along the propagation path.

Finally, we solve for the short stationary waves or the inertial boundary layers for those cases where $\gamma \sim O(1)$ or less. For simplicity the small $O(\epsilon)$ forcing term will be neglected as it only makes a small frictional correction. In the heating region the large α part of (40) gives

$$\frac{i}{k}\frac{\partial q_{n+1}}{\partial x} + \gamma_n q_{n+1} = -(2k^2\Delta)\cos kx \,\frac{Q_{n-1} + (n+1)Q_{n+1}}{1 + \Delta/c_n} \tag{58}$$

where $\gamma_n = \epsilon/(c_n + \Delta)k$, $c_n = 1/(2n+1+2k^2)$

a particular solution of which is

$$q_{n+1} = -\frac{2k^2\Delta}{1+\gamma_n^2}\left(\gamma_n \cos kx + \sin kx\right)\frac{Q_{n-1} + (n+1)Q_{n+1}}{1 + \Delta/c_n} \tag{59}$$

and v_n and π_{n-1} are found from (49)

To make the flow continuous we add stationary waves if $\Delta > 0$,

$$\begin{aligned} q_{n+1} &= S\cos\left[k_n(x+\xi)\right]\exp\left[-\epsilon(x+\xi)/2\Delta(1-(2n+1)\Delta)\right] \\ q_{n+1} &= T\sin\left[k_n(x+\xi)\right]\exp\left[-\epsilon(x+\xi)/2\Delta(1-(2n+1)\Delta)\right] \end{aligned} \tag{60}$$

and v_n and π_{n-1} are found from (49). Since the waves propagate eastwards $q = r = v = 0$ for $x < 0$. To make them continuous at $x = 0$ we must take

$$S, T = \left(\gamma_n, \frac{k}{k_n}\right)\frac{2k^2\Delta}{1+\gamma_n^2}\,\frac{Q_{n-1} + (n+1)Q_{n+1}}{1 + \Delta/c_n} \quad , \quad \xi = 0 \tag{61}$$

At $x = 2\pi/k$ we also use (60), but with $\xi = -2\pi/k$ and minus the amplitude in (61).

If $\Delta < 0$ we have two inertial boundary layers

$$q_{n+1} = E \exp\left[k_n'(x+\xi)\right] \quad , \quad q_{n+1} = F \exp\left[-k_n'(x+\xi)\right] \tag{62}$$

and once again v and π are found from (49). Consider the matching at x = 0. For x < 0 we have the first of (62), and for x > 0 we have (59) and the second of (62). Continuity at x = 0 then requires

$$E, F = \left(\gamma_n + \frac{k}{k_n'} \, , -\gamma_n + \frac{k}{k_n'}\right) \frac{-k^2\Delta}{1+\gamma_n^2} \; \frac{Q_{n-1} + (n+1)Q_{n+1}}{1+\Delta/C_n} \, , \quad \xi = 0 \tag{63}$$

At x = $2\pi/K$ we have (62) but with $\xi = -2\pi/K$ and minus the amplitude in (63). when $\gamma_n^2 >> 1$ the stationary waves cannot propagate, so that the $O(\Delta)$ forcing terms make only a small contribution to the trapped response (54) and may therefore be neglected.

6. Results

We now have a model with five parameters: the length, width, and latitude of the heating region, the damping time constant, and the magnitude of the zonal mean wind. We wish to illustrate the effects of these parameters, but the number of cases that can be presented is limited. We shall therefore proceed by varying the parameters one at a time, keeping the others constant.

The most important parameter is perhaps the damping time constant. In figures one and two we illustrate two different cases, one with a broad heating region centred on the equator, and one with a narrow heating region centred at $y_o = 1$. In both cases we use values of ϵ of 0.3, 0.1, and 0.05, corresponding to damping times of somewhat less than one day, two and a half days, and five days. The very large value of $\epsilon = 0.3$ was found by Holton and Colton (1972) to give the best results when trying to reproduce to observed streamfunction using a linear model.

In both cases we find that a number of important features are found for all values of ϵ. when the heating is symmetric about the equator, at the surface there is a flow into the heating region from the west which is stronger but narrower than the inflow from the east. In the middle of the heating region the flow turns poleward and then westward. For all values of ϵ the pressure is lowest at the equator on eastern side, and at approximately $y = \pm 1.5$ on the western side. In the case of the narrow heating region centered at $y_o = 1$, the minimum pressure is found to the west and slightly poleward of the heating maximum. The strongest inflow is from the south-west and is located north of the equator.

The major difference between the solutions with different damping values is the zonal extent of the flow. This may be explained in terms of the wave propagation properties. If one thinks of the heating as being switched on at t = 0, then the Kelvin wave propagates east at speed c, and the fastest planetary wave propagates west at speed c/3. The region affected by the heating increases at a rate 4c/3. With c $\sim$ 50m/s this amounts to about 6000km/day (six units of x). Thus with a time constant of one day(ϵ=0.3) the extent of the flow is of the order of 6000 km, 15000km for two and a half days, 30000km for five days, in agreement with figures one and two. Thus we see that with a time constant of one day the response is localised, while for time constant of a week or more one needs a global model. We shall mainly consider the intermediate case ϵ =0.1. Note that in all cases presented in this paper, except the last one, the total amount of heating is the same. All cases are therefore directly comparable, in the sense that the amount of forcing is the same.

In figure 3 we show the effect of changing the width of the heating region, the widths being in the ratio 1, 2/3, and 1/3 of the width of $D_0(y)$. As the width is reduced, so is the poleward flow in the heating region. In the case of the narrowest heating region (α=3) the Kelvin mode flow appears to merge smoothly with the planetary wave flow. The explanation for the disappearance of the poleward flow is that as the heating becomes concentrated on the equator the vortex stretching mechanism becomes weaker, because f is smaller.

In figure 4 we have a narrow heating region (α=3) centered at y_o = 1, the heating being spread over different lengths (2π, 4π, 6π). The most striking feature is that the length of the flow region does not change much. Moreover, the meridional velocities drop rapidly as the length of the heating region increases, the zonal velocities being reduced somewhat less. The fact that the length of the region with significant flow does not change much is easily explained. As the length of the heating region increases, so the Kelvin wave emitted from its western end, and the planetary waves emitted from its eastern end are more heavily damped by the time they reach the other end of the heating region. The reduction in the poleward flow is partly due to the reduction of Q and w at each point, reducing the vortex stretching. There is also more overlap with the planetary waves coming from eastern boundary, which reduces v even further. The low off the equator then also becomes weaker.

Figure 5 shows a narrow heating region (α=3) being moved from the equator to y_o =1, and then y_o =2. As discussed previously, when the heating is centered on the equator the vortex stretching mechanism is ineffective and there is zonal flow past the heating region. As the heating moves to y_o =1, and then to y_o =2 the vortex stretching mechanism becomes more and more effective, and the poleward flow in the heating region increases correspondingly. And as the poleward flow increases so - by the geostrophic relation - a low to the west of it develops. As the heating moves away from the equator its projection onto $D_o(y)$ decreases, and the Kelvin wave becomes weaker.

We now examine the effects of a zonal mean wind. Figure 6 shows the flow associated with a broad heating region ($Q \propto D_o(y)$) centered on the equator. Graphs of ϕ and u along the equator are shown in fig. 7 and 8. In a positive (westerly) mean wind the cyclonic circulation is stronger than in still air. The planetary waves are stronger but decay faster. Little change can be seen in the amplitude or the decay rate of the Kelvin wave, but it has developed wiggles. They are the stationary planetary waves. In a negative (easterly) mean wind the main change is that the amplitude and the decay rate of the planetary wave are both considerably reduced. All these effects are in accordance with the results of Lau and Lim (1982) and Lim and Chang (1983).

Noting that, while visually quite noticeable, the stationary short waves are very weak quantitatively, we shall try to explain the changes in the long waves. The changes in the decay rates of the waves may be explained in terms of their group velocities. In a westerly wind the long planetary waves propagate against the wind and therefore see their velocity reduced, while the velocity of the Kelvin wave is increased. But as the velocity of the Kelvin wave is much larger than that of the planetary waves, it is much less affected by a small mean wind than they are. And as the spatial decay rate of each wave mode is inversely proportional to its group velocity, it follows that the long planetary waves will decay faster in a westerly wind, and more slowly in an easterly wind.

Let us now try to explain the changes in the amplitudes of the long waves. At the equator the zonal momentum equation may be written

$$\frac{\partial}{\partial x}(\phi+\Delta u) = -\epsilon u \tag{64}$$

But $(\phi + \Delta u)$ is just the linearised form of the Bernoulli constant $H = \phi + 1/2\ (u^2 + v^2 + w^2)$. So (64) says that the changes in the Bernoulli function are proportional to the frictional losses. But if the heating region is short compared to the two wave decay regions (γ is small) then the change in H across it will be small compared to the change in H in the wave decay regions. Thus in the small γ limit the changes in H in the two wave regions are equal and opposite, so that

$$\int_{-\infty}^{0} u_W\, dx = -\int_{2\pi/k}^{\infty} u_E\, dx \tag{65}$$

But the integral $\int u dx$ is proportional to the amplitude A of the wave, and to its group velocity $(C+\Delta)$. Therefore

$$(C_W+\Delta)A_W = -(C_E+\Delta)A_E \tag{66}$$

But if $Q \propto D_o(y)$ then $C_E=1$, $C_W=-1/3$

$$\frac{A_W}{A_E} = 3\left(\frac{1+\Delta}{1-3\Delta}\right) \tag{67}$$

Thus the ratio of the amplitudes of westerlies to easterlies increases from 3 in still air to 9 for $\Delta=0.2$, and decreases to 1.5 for $\Delta=-0.2$.

Plots of the Bernoulli function along the equator are shown in figure 9. We see that its maximum value is hardly affected by Δ in this case, whilst its rate of change on the eastern and western sides is affected by the mean wind. Note that since the short stationary waves are oscillatory $\int u dx$ is small and the above argument is not seriously affected.

Figure 10 illustrates the effect of the mean wind on the flow associated with a narrow heating region ($\alpha = 3$) centered at $y_0 = 1$. As before, the amplitude and the decay rate of the planetary modes are increased in a westerly mean wind, the opposite holding in easterly mean wind. What is noteworthy here is that even though the heating is almost entirely north of the equator it produces strong flows in the other hemisphere, especially in the presence of a westerly mean wind.

The case in figure 11 is identical to that in figure 10 except that the heating is spread out over a larger range of longitudes (4π instead of 2π). For $\Delta = 0$ and $\Delta = -0.2$ the flows in figures 10 and 11 are very similar, but for $\Delta = 0.2$ the two figures differ significantly. In the case of the larger heating region the effects of the westerly mean wind are milder, and the short stationary waves are no longer visible. This is due both to their amplitude being four times smaller (as it is proportional to K^2) and to the two ends of the heating region being further apart.

The final case shown in figure 12 is rather different in that we use the strong damping ($\epsilon = 0.3$) in combination with a long ($K = 1/3$) narrow ($\alpha = 3$) heating region. All three choices contribute towards causing large γ's. The result is a flow with most of the features of the classical two-dimensional Hadley circulation model. When the heating is centred on

the equator we have northeasterly and southeasterly "trade winds" to the north and south. The sinking occurs poleward of the heating region. With the heating moved to $y_0 = 1$ we see the southeasterly trades turn into southwesterlies as they cross the equator. The strongest flows are now on the equatorward side of the heating region, and most of the sinking takes place across the equator.

The main difference with the purely two dimensional models arises from the fact that the Kelvin mode is not trapped. As a result we have some easterlies at the eastern end of the heating region, and westerlies at the western end. In a purely two dimensional model, on the other hand, the zonal velocity would be exactly zero at the equator. The two results are perfectly compatible since the integral of (64) along the equator gives

$$\int_{-\infty}^{\infty} u(x,0)\,dx = 0 \qquad (68)$$

as ϕ, u ---> 0 as x ---> $\pm\infty$. The zonal integrals of the easterlies and westerlies are equal and opposite, so that in a zonally averaged sense u is zero on the equator.

Appendix

We want to write the function $D_0[\alpha(y-y_0)]$ in terms of the parabolic cylinder functions $D_n(y)$,

$$D_0\left[\alpha(y-y_0)\right] = \sum_{n=0}^{\infty} C_n D_n(y) \tag{A.1}$$

Then

$$\int_{-\infty}^{\infty} D_0\left[\alpha(y-y_0)\right] D_n(y)\, dy = C_n \int_{-\infty}^{\infty} D_n^2(y)\, dy \tag{A.2}$$

by the orthogonality relation

$$\int_{-\infty}^{\infty} D_n D_m\, dy = 0 \quad \text{if } n \neq m \tag{A.3}$$

We start with the identity

$$\frac{d}{dy}\left\{D_0\left[\alpha(y-y_0)\right] D_n(y)\right\} = D_0\left[\alpha(y-y_0)\right]\left\{D_n' - \tfrac{1}{2}\alpha^2(y-y_0)D_n\right\} \tag{A.4}$$

If we integrate (A.4) we have

$$\int_{-\infty}^{\infty} D_0\left[\alpha(y-y_0)\right]\left\{D_n' - \tfrac{1}{2}\alpha^2(y-y_0)D_n\right\} dy = 0 \tag{A.5}$$

If we now use (7) and (A.2) in (A.5) we obtain

$$(1+\alpha^2) C_{n+1} \int_{-\infty}^{\infty} D_{n+1}^2\, dy = \alpha^2 y_0 C_n \int_{-\infty}^{\infty} D_n^2\, dy + (1-\alpha^2) n C_{n-1} \int_{-\infty}^{\infty} D_{n-1}^2\, dy \tag{A.6}$$

but since

$$\int_{-\infty}^{\infty} D_n^2 \, dy = \sqrt{2\pi}\; n! \tag{A.7}$$

we find

$$c_{n+1} = \frac{1}{n+1}\left[\left(\frac{1-\alpha^2}{1+\alpha^2}\right) c_{n-1} + \left(\frac{\alpha^2}{1+\alpha^2}\right) y_o \, c_n\right] \tag{A.8}$$

Now to find c_o we use (A.2) with n=0 together with

$$\int_{-\infty}^{\infty} D_o(\gamma y)\, dy = \sqrt{2\pi}/\gamma \tag{A.9}$$

and find that

$$c_o = \left(\frac{2}{1+\alpha^2}\right)^{1/2} \exp\left(\frac{-1}{4}\frac{\alpha^2}{1+\alpha^2} y_o^2\right) \tag{A.10}$$

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Figure Captions

Figure 1. The flow at the surface (wind arrows) and the pressure contours (broken lines, contour interval of 0.5) for a broad heating region centred on the equator (thick solid line, K=1, α=1, y_o=0, amplitude = 0.5) The zonal extent of the flow outside the heating region increases rapidly as the damping is decreased from ϵ=0.3 (a), to ϵ=0.1(b), and ϵ=0.05(c). The x and y coordinates are in units of equatorial Rossby radii, a $\sim$ 1000km.

Figure 2. The flow at the surface and the pressure contours for a narrow heating region centred off the equator (K=1, α=3, y =1, amplitude = 0.5) as the damping is reduced from ϵ = 0.3 (a), to ϵ=0.1 (b), and ϵ=0.05 (c). Conventions and contours as in fig. 1.

Figure 3. The surface winds and pressure for a heating region centered on the equator (K=1, y =0, amplitude = 0.5, ϵ=0.1) as the meridional extent of the heating is reduced: α=1 in (a), α=1.5 in (b), and α=3 in (c).

Figure 4. The surface winds and pressure for a narrow heating region off the equator (α=3, y =1, amplitude = 0.5, ϵ=0.1) as the length of the heating region is increased: K=1 in (a), K=0.5 in (b), and K=0.33 in (c).

Figure 5. The surface winds and pressure for a narrow heating region (K=1, α =3, amplitude = 0.5, ϵ =0.1) being moved off the

equator: $y_o = 0$ in (a), $y_o = 1$ in (b), and $y_o = 2$ in (c).

Figure 6. The surface winds and pressure for a broad heating region centred on the equator ($K=1$, $\alpha=1$, $y=0$, amplitude = 0.5, $\epsilon=0.1$) in the absence of mean wind, $\Delta=0$, (a), in the presence of a westerly mean wind, $\Delta=0.2$, (b), and in the presence of an easterly mean wind, $\Delta=-0.2$, (c).

Figure 7. The surface pressure ϕ along the equator for the case shown in figure 6, in the absence of mean winds (a), in the presence of a westerly mean wind $\Delta=0.2$ (b), and in the presence of an easterly mean wind $\Delta=-0.2$ (c).

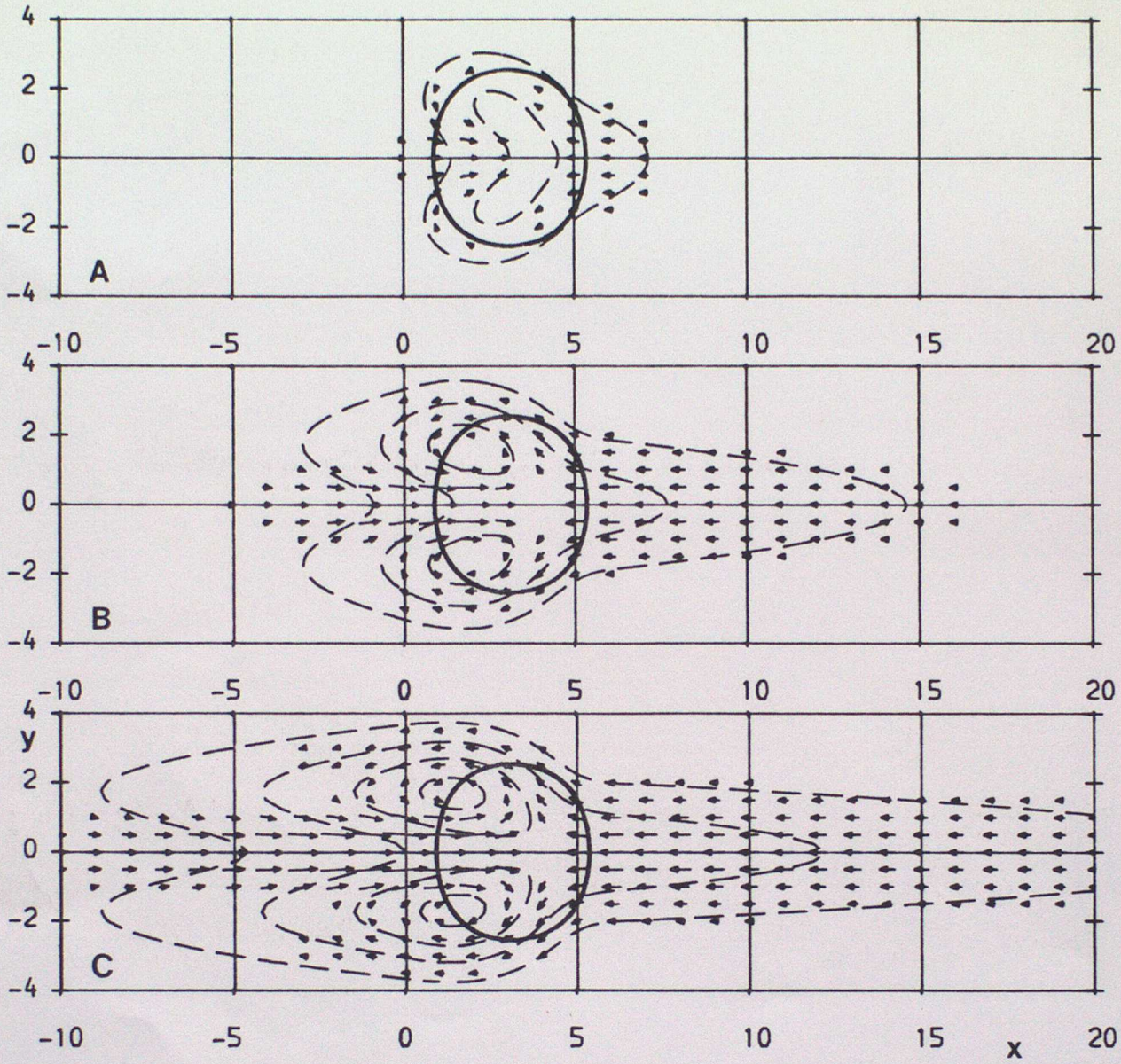
Figure 8. The surface zonal velocity u along the equator for the case shown in figure 6, in the absence of mean winds (a), in the presence of a westerly mean wind $\Delta=0.2$ (b), and in the presence of an easterly mean wind $\Delta=-0.2$ (c).

Figure 9. The surface Bernoulli constant $(\phi+\Delta u)$ along the equator for the case shown in figure 6, in the absence of mean winds (a), in the presence of a westerly mean wind $\Delta = 0.2$ (b), and in the presence of an easterly mean wind $\Delta = -0.2$ (c).

Figure 10. The surface winds and pressure for a narrow heating region centred off the equator ($K= 1, \alpha= 3$, $y_o = 1$, amplitude = 0.5, $\epsilon= 0.1$) in the absence of mean winds (a), in the presence of a westerly mean wind $\Delta= 0.2$ (b), and in the presence of an easterly mean wind $\Delta = -0.2$ (c).

Figure 11. The surface winds and pressure for a narrow heating region centred off the equator ($K = 0.5, \alpha = 3, y_o = 1$, amplitude= $0.5, \epsilon = 0.1$) as in figure 10 except that the heating is spread over twice the range of longitudes. Flow in the absence of mean winds (a), in the presence of a westerly mean wind $\Delta = 0.2$ (b), and in the presence of an easterly mean wind $\Delta = -0.2$ (c).

Figure 12. The surface winds, pressure (a,c), and vertical velocity (b,d) for a long narrow heating region and strong damping ($K = 0.33, \alpha = 3$, amplitude= 1, $\epsilon = 0.3$). The heating is centred on the equator in (a,b), and on $y_o = 1$ in (c,d). The vertical velocity contours in (b,d) are drawn at intervals of 0.2 : solid lines for rising motion, dotted lines for no motion, and broken lines for sinking motion.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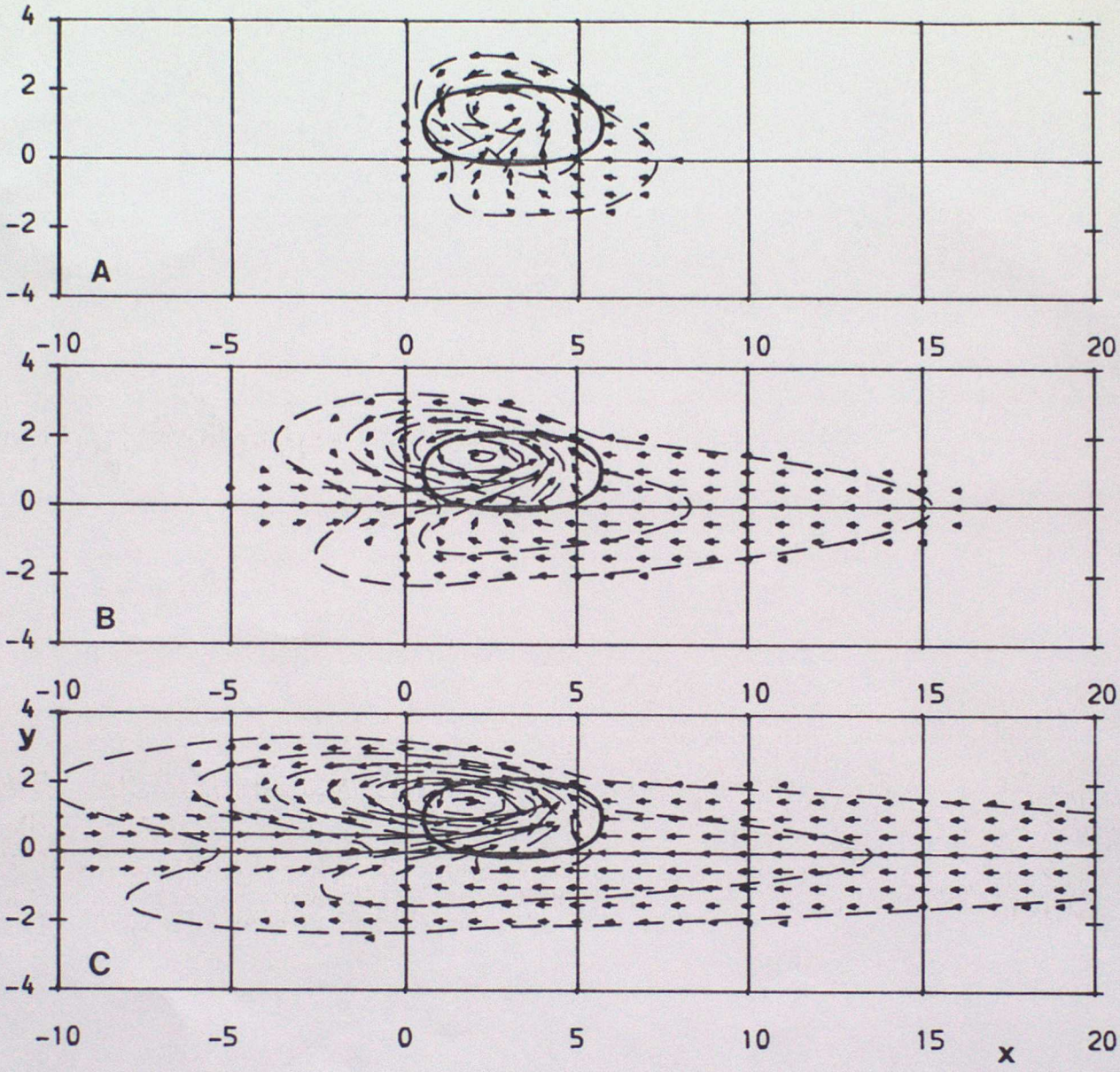
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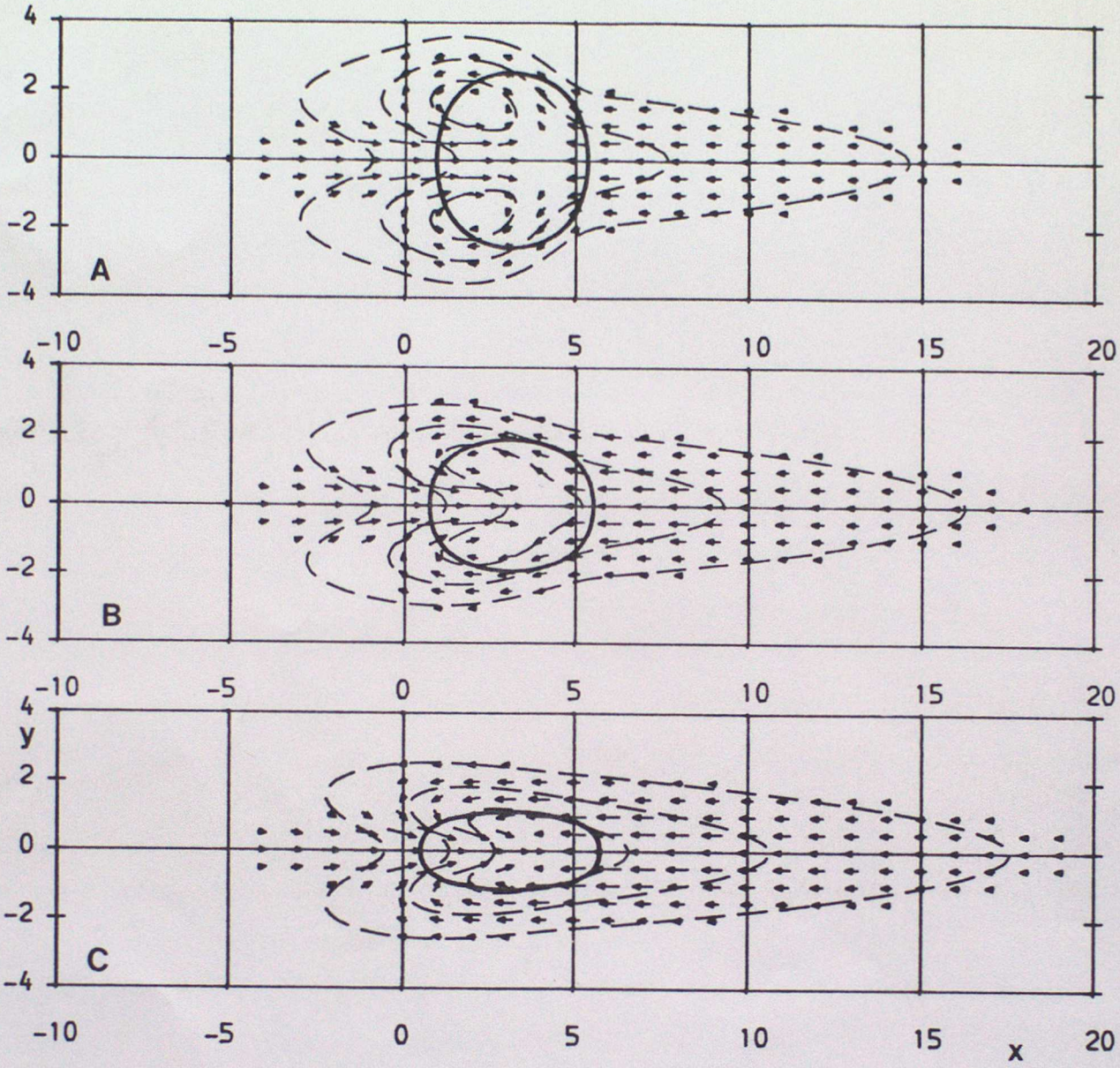
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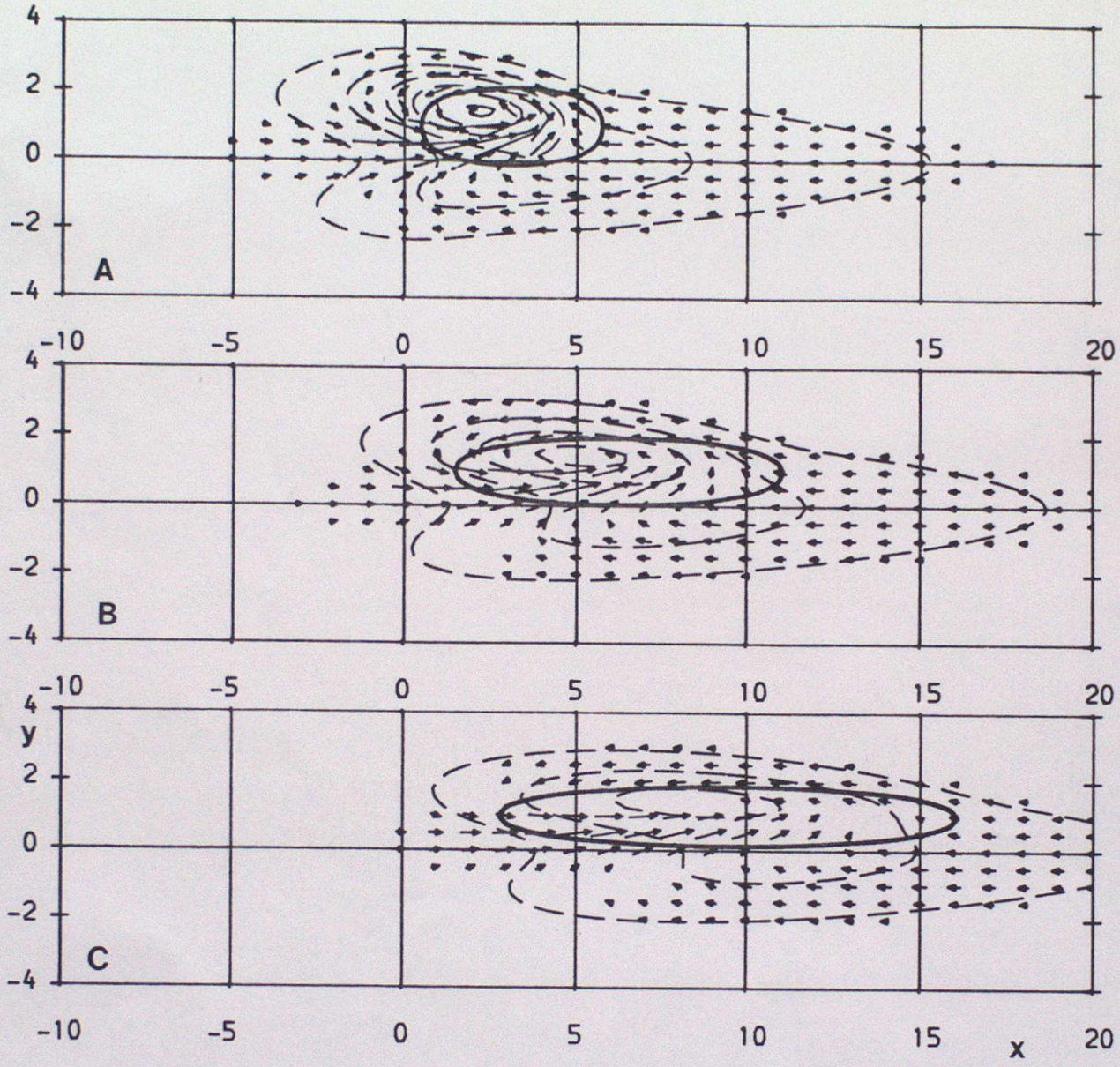
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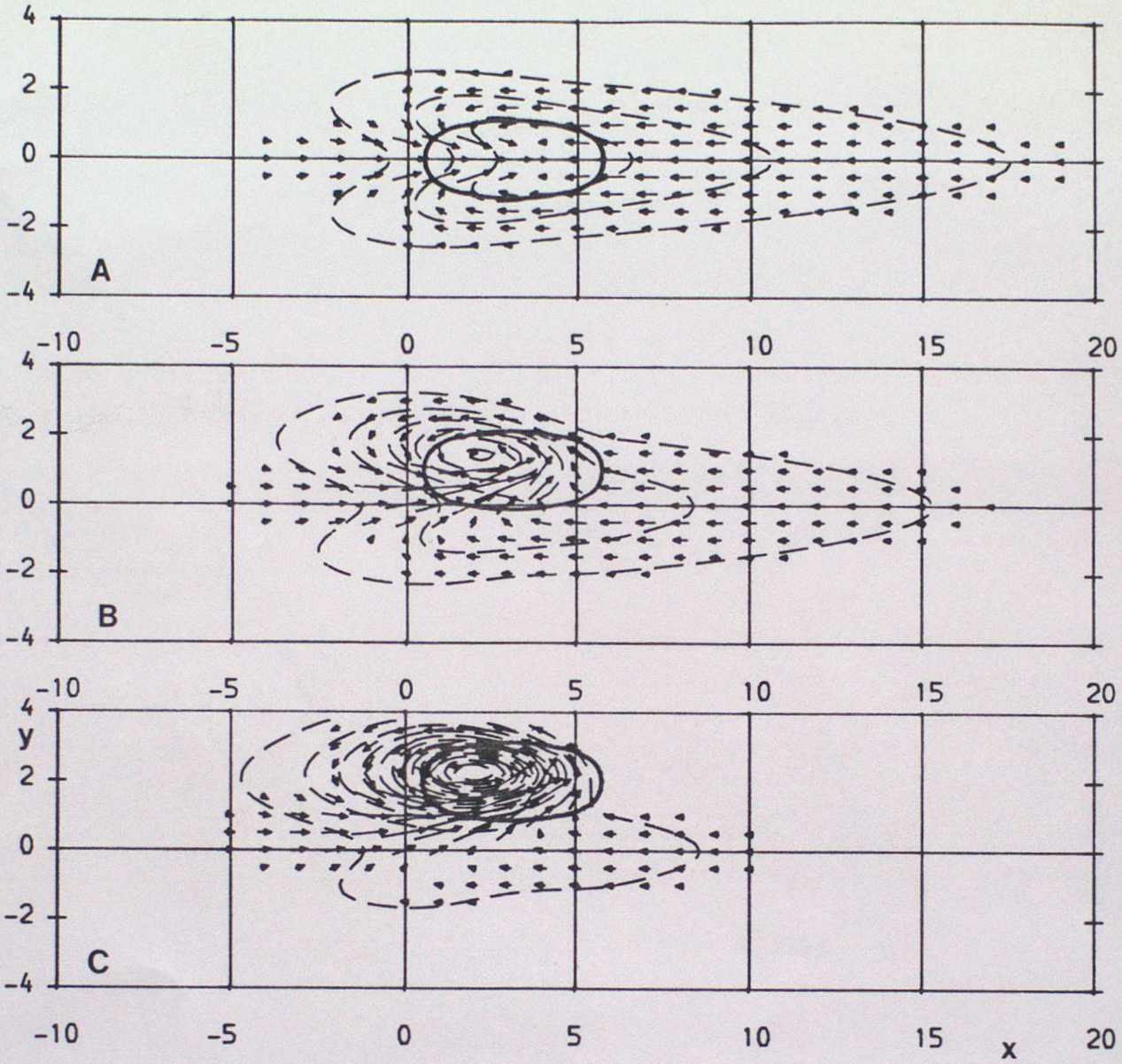
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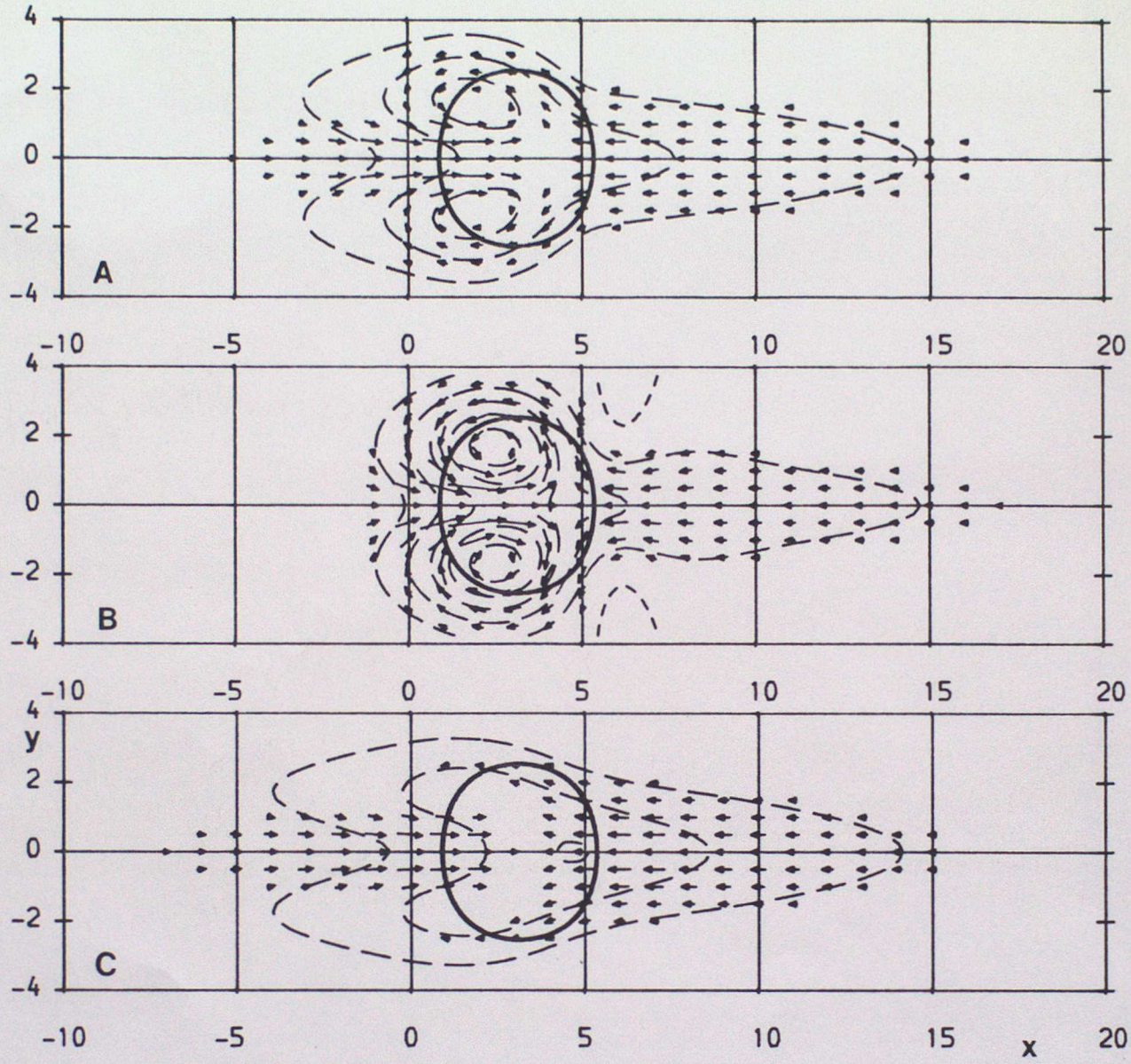
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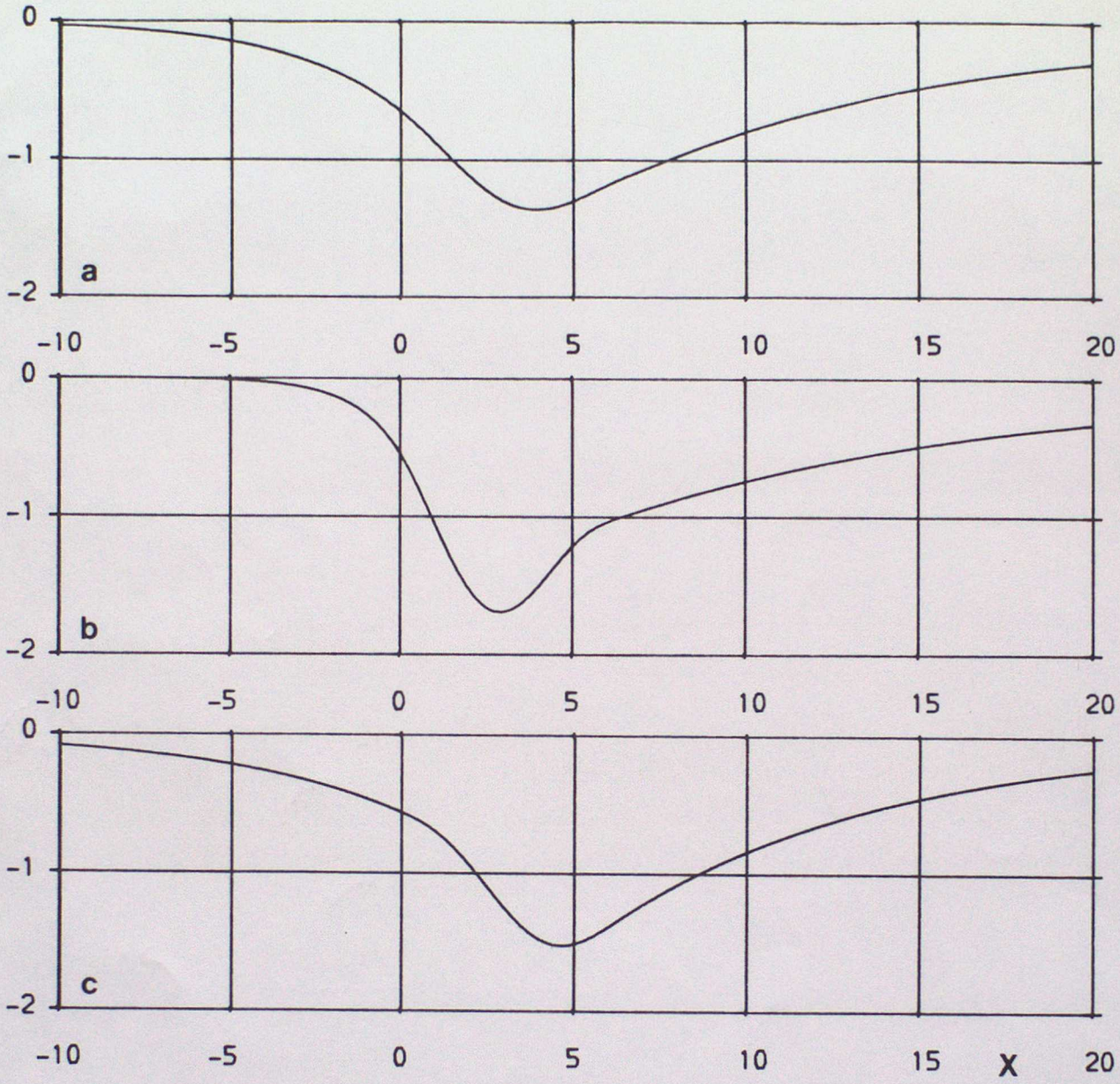
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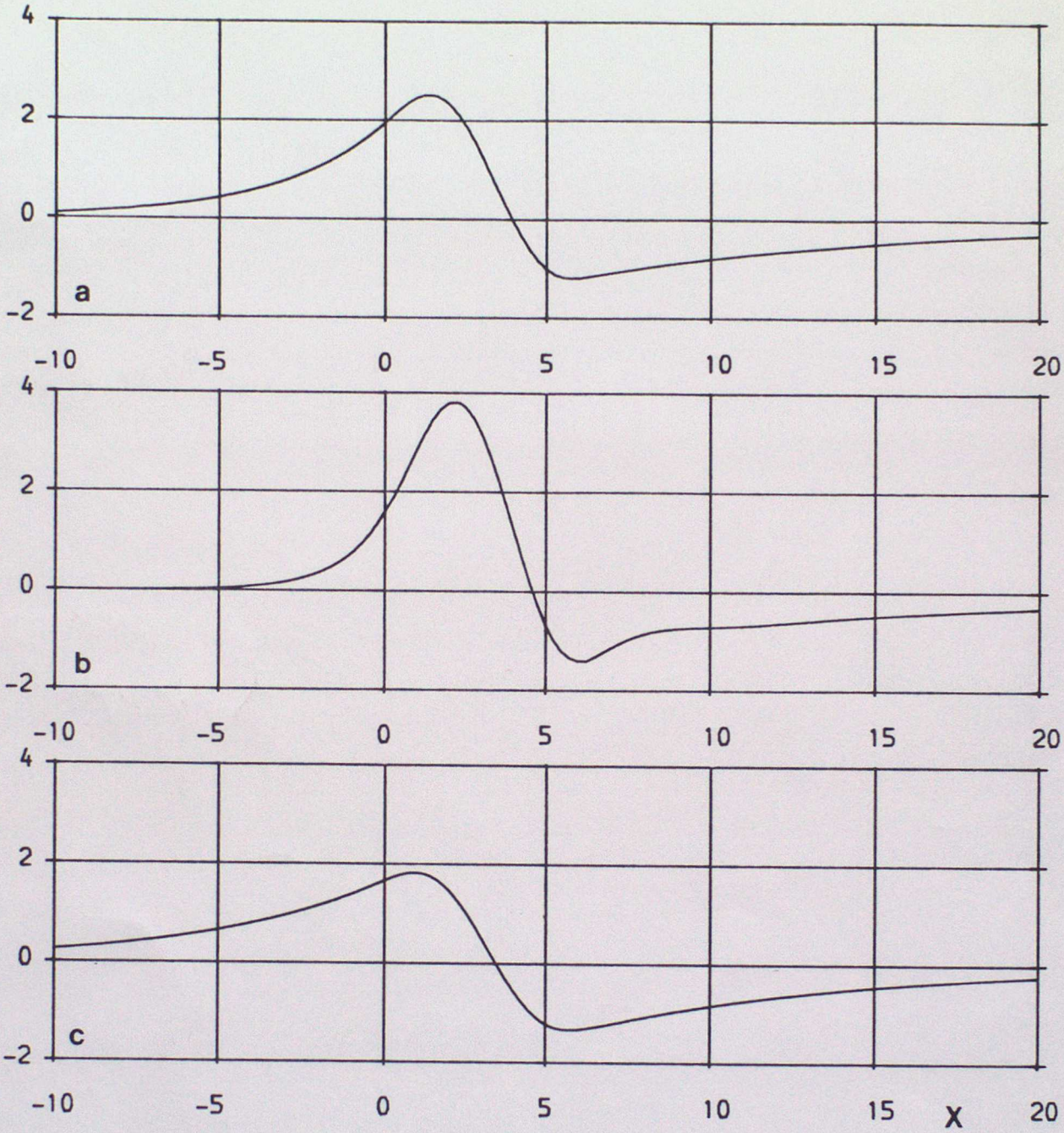
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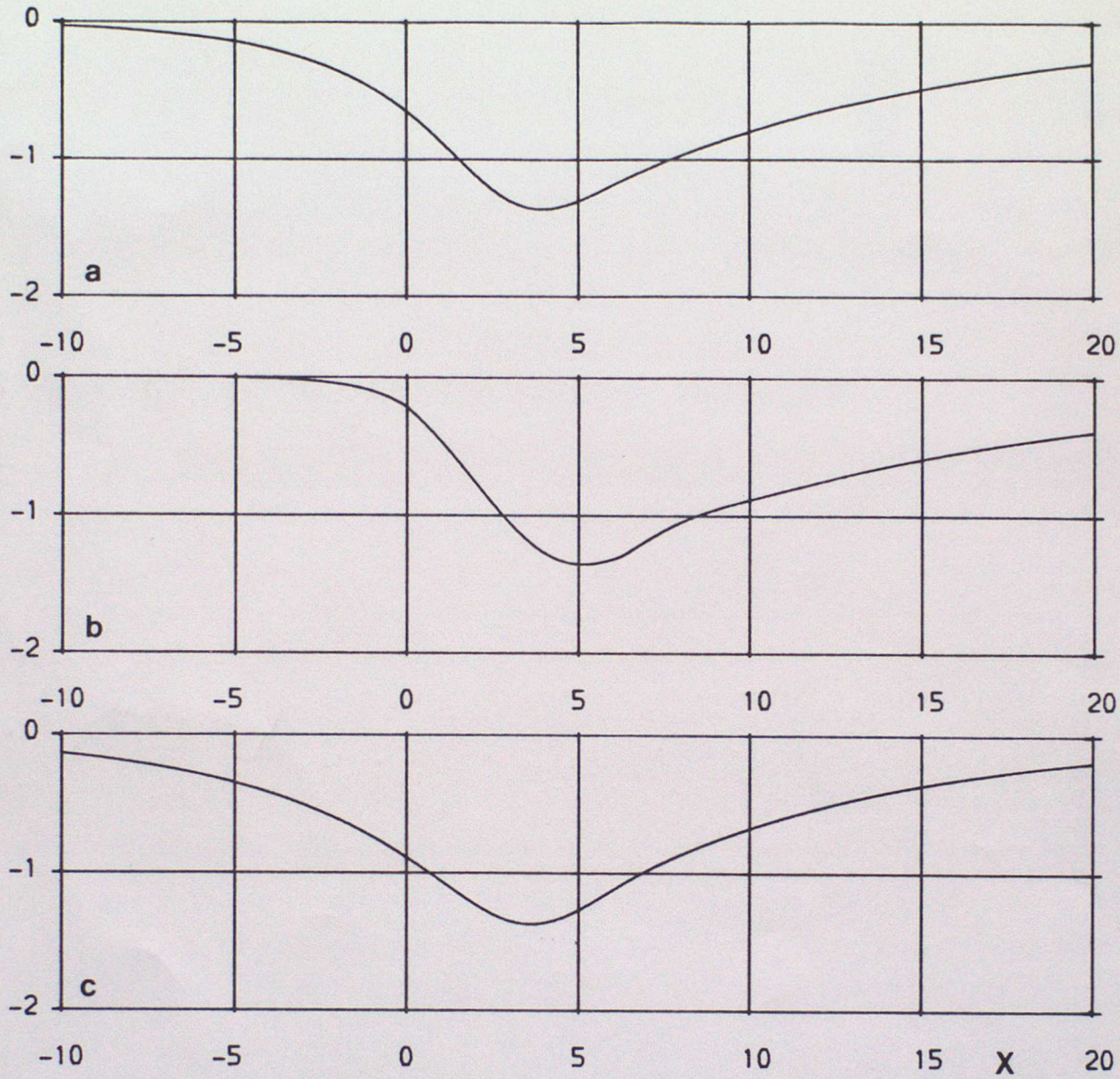
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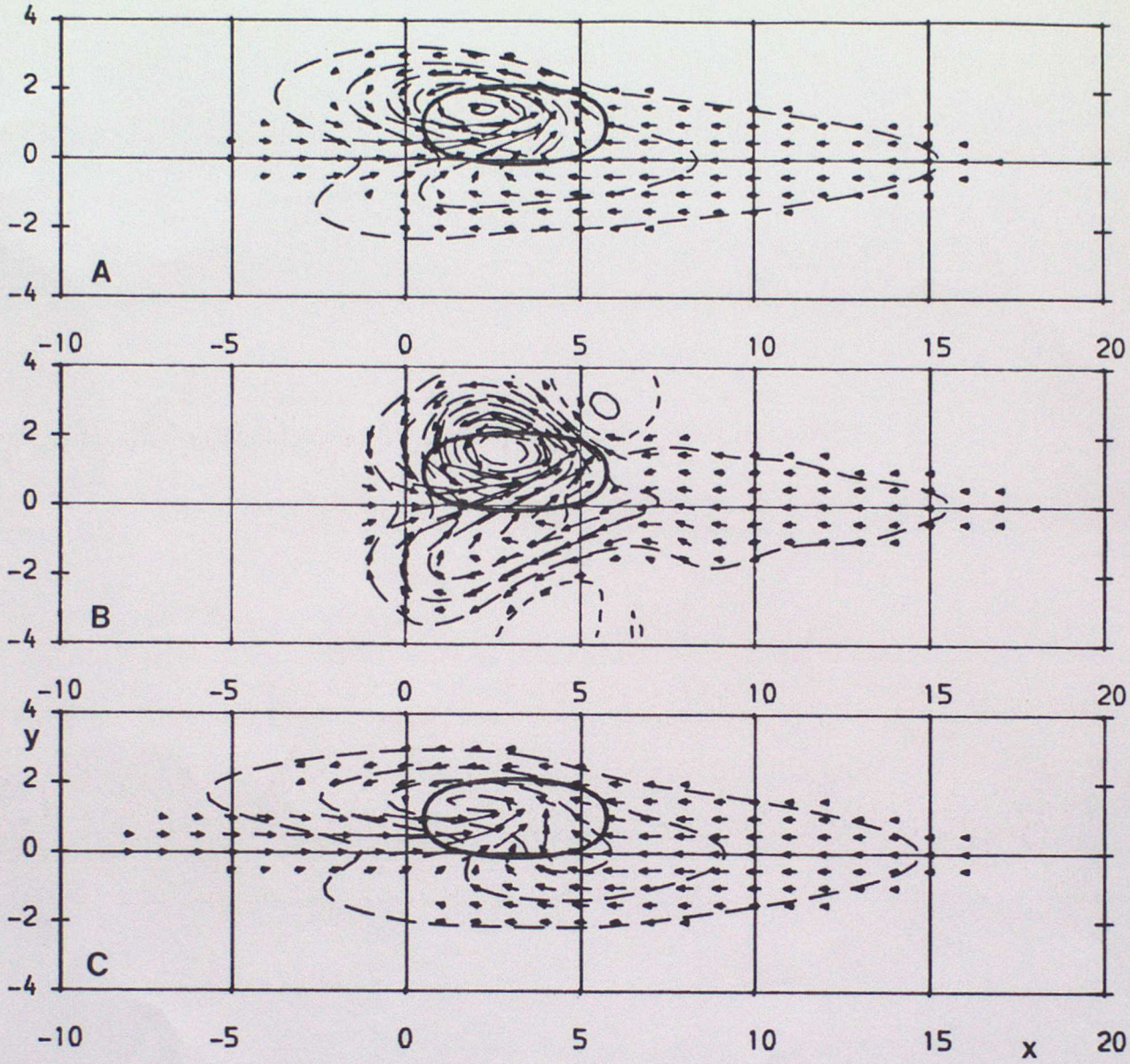
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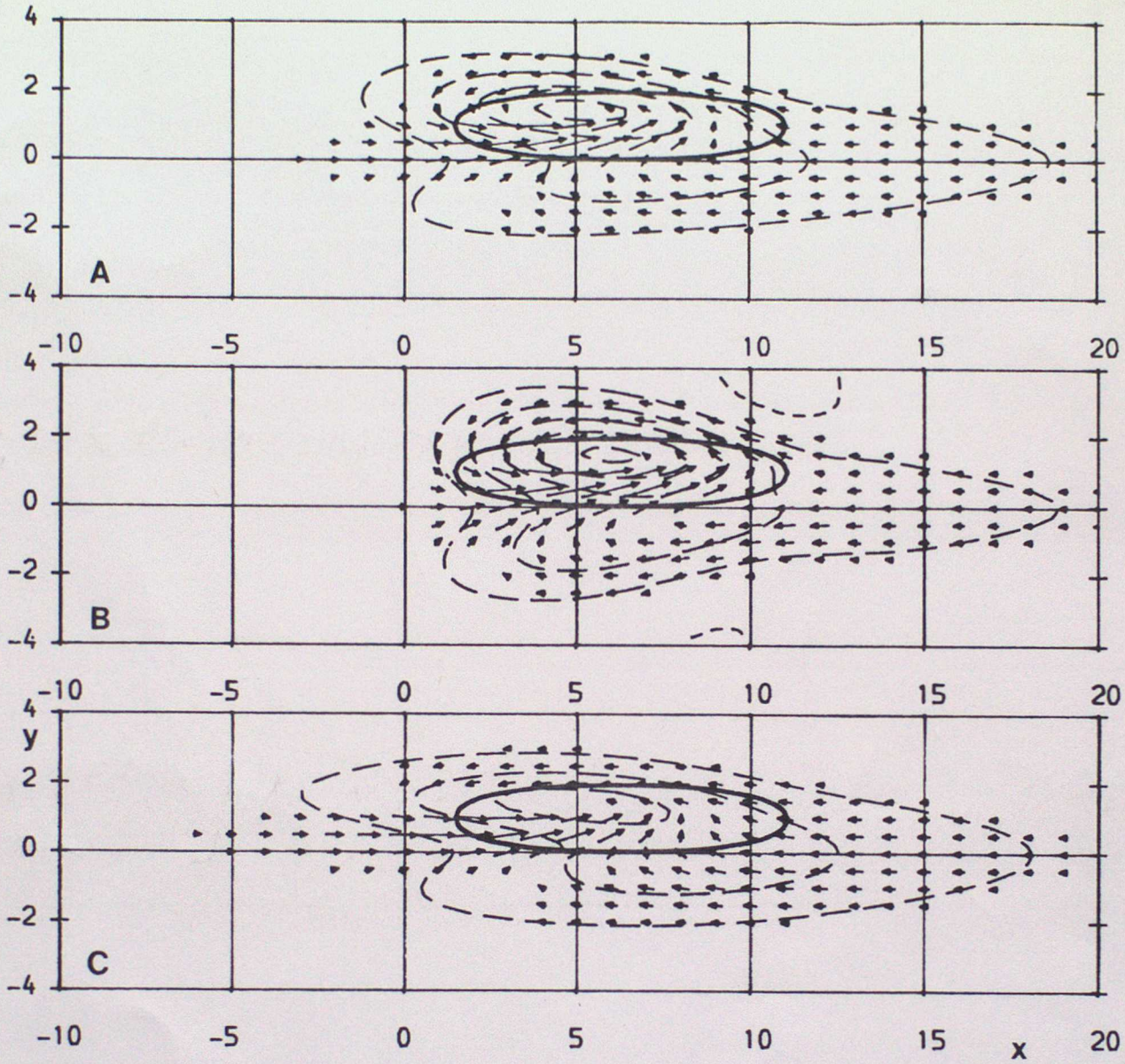
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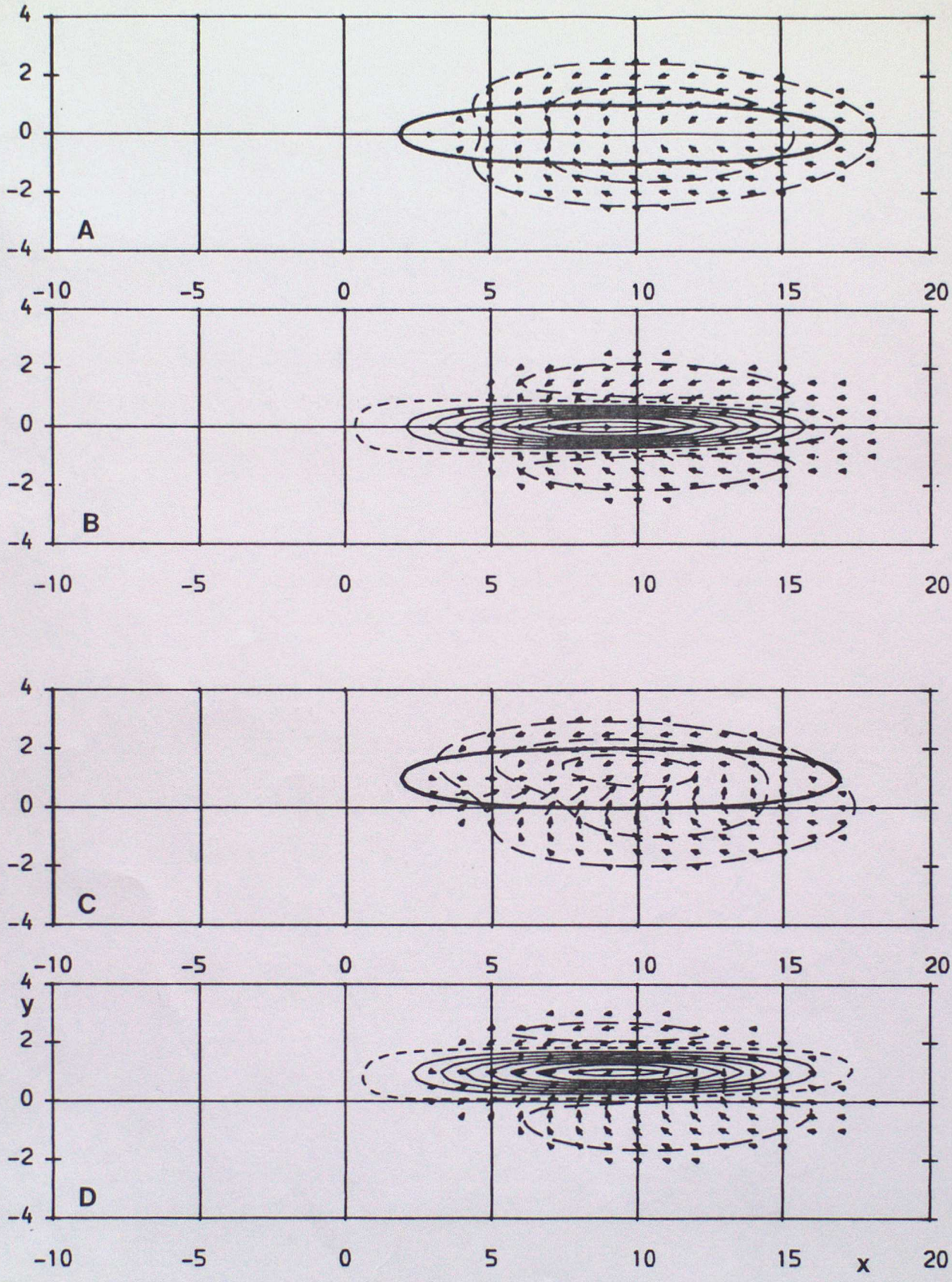
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DCTN 1	June 1984	A scheme for incorporating the thermodynamic sea-ice model into a coupled ocean-atmosphere model. C Gordon and M Bottomley
DCTN 2	Sept 1984	The parametrization of the upper ocean mixed layer in coupled ocean/atmosphere models. C Gordon and M Bottomley
DCTN 3	Sept 1984	The response of a simple thermodynamic sea-ice model to GCM forcing appropriate for perennial sea-ice. M Bottomley
DCTN 4	Sept 1984	Potential vorticity in the stratosphere derived using data from satellites. S A Clough, N S Grahame and A O'Neill
DCTN 5	Sept 1984	Simulation of the seasonal cycle of SST using globally specified climatological data to force simple ocean models. N S Grahame
DCTN 6	Sept 1984	Development of an improved GCM radiation scheme. A Slingo and R C Wilderspin
DCTN 7	Sept 1984	On the specification of surface fluxes in coupled atmosphere/ocean general circulation models. J F B Mitchell, C A Wilson and C M Price
DCTN 8	Sept 1984	A survey of possible repercussions of the Soviet river reversal programme on Arctic sea-ice. H Cattle
DCTN 9	Oct 1984	A study of a strong stratospheric warming during January 1982 T D A Fairlie
DCTN 10	Nov 1984	An intercomparison of the Met O 11 N45 model and the Met O 20 11-level model on the Cyber 205 J F Dyson
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DCTN 13	Nov 1984	On modelling the effects of CO_2 on climate. J F B Mitchell
DCTN 14	Dec 1984	Development of a revised longwave radiation scheme for an atmospheric general circulation model. A Slingo and R C Wilderspin
DCTN 15	Jan 1985	Rossby wavetrains in the stratosphere forced by localised disturbances in the troposphere. C J Marks (Oxford), A O'Neill and V D Pope

DYNAMICAL CLIMATOLOGY TECHNICAL NOTES

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DCTN 29	Sept 1985	Handbook of the Meteorological Office 11-layer atmospheric general circulation model. Volume 1 : Model Description Edited by A Slingo
DCTN 30	Sept 1985	Impact of land surface variations on African rainfall in general circulation models. P R Rowntree, M F Wilson and A B Sangster